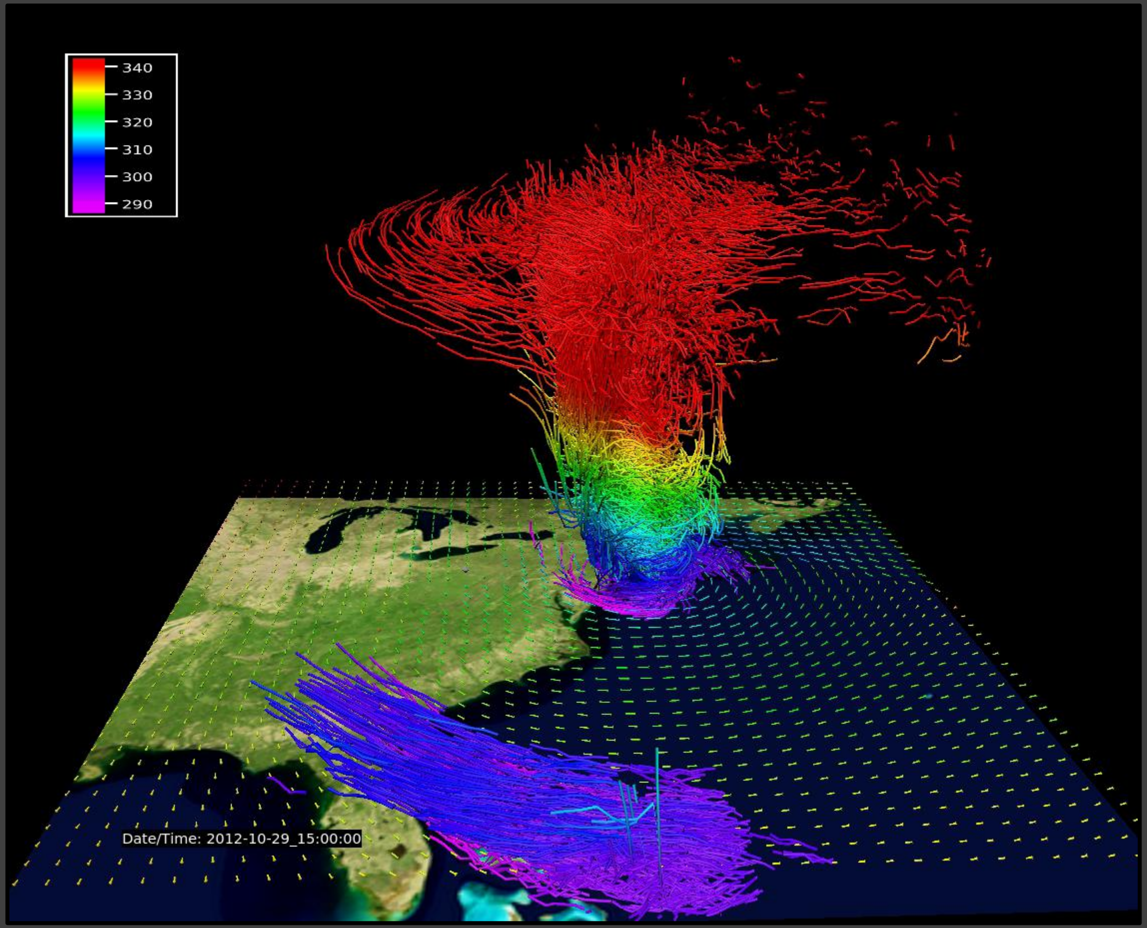
Making Sense of Numerical Weather Prediction (NWP)

Computational Fluid Dynamics (CFD) and Model Physics

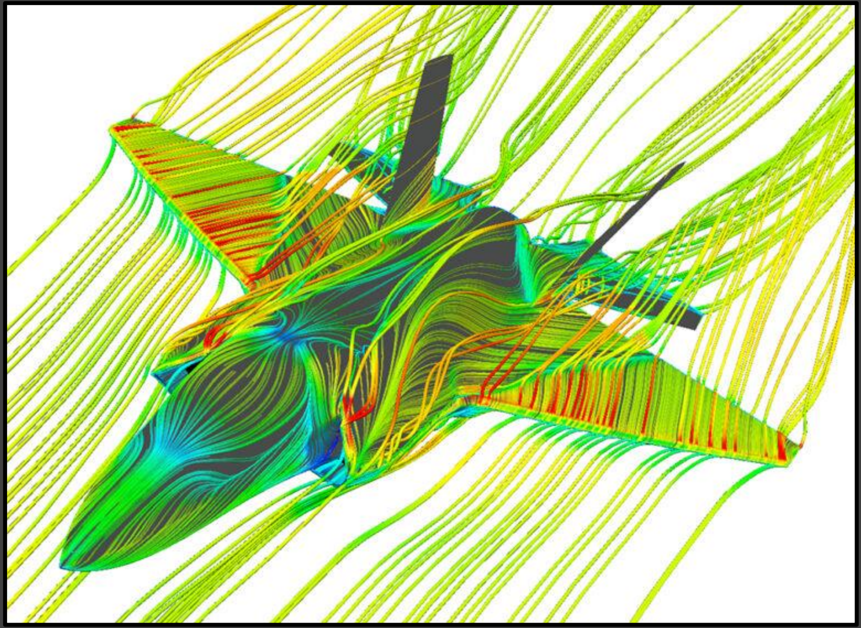
NWP Models

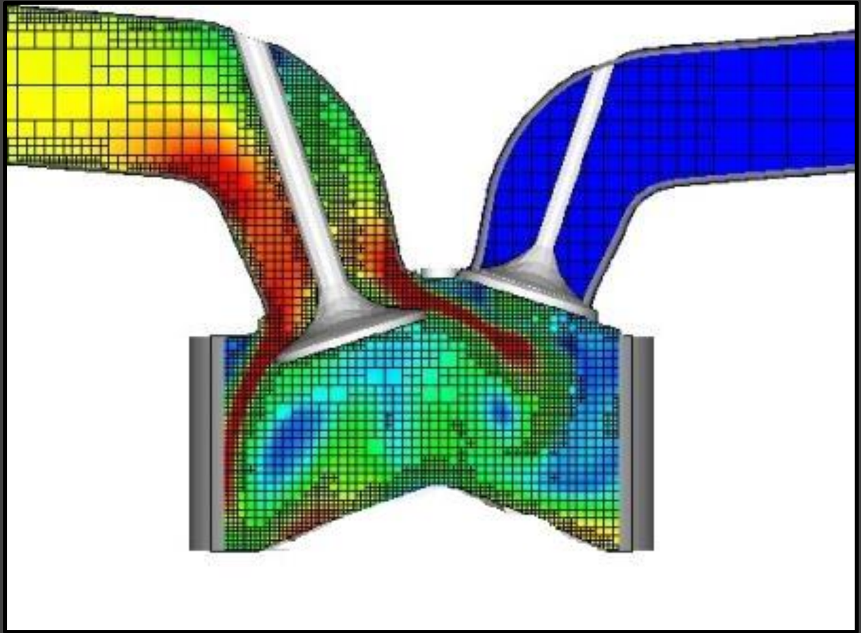
Computational fluid dynamics (CFD) model coupled with models representing various physical processes that have a significant effect on atmospheric dynamics



Navier-Stokes Equations

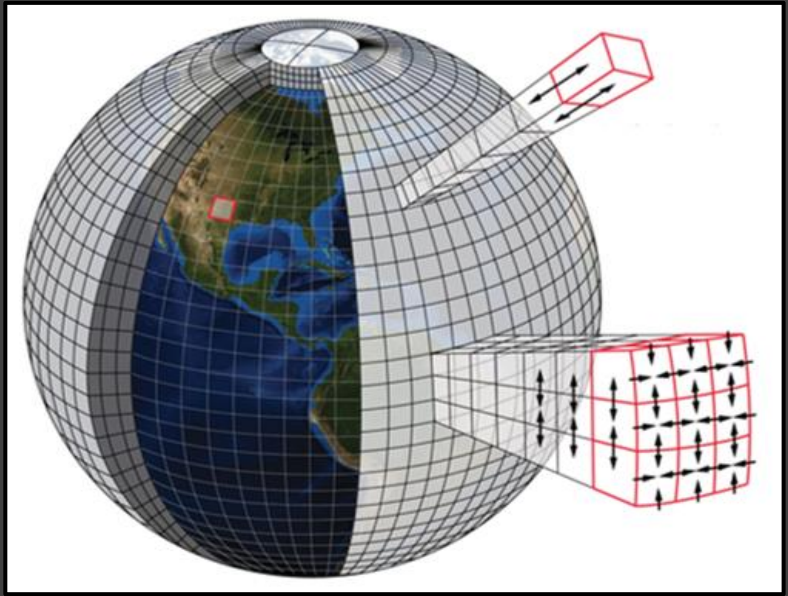
- Set of partial differential equations (PDE) describing the time varying flow of viscous fluids
- Think "how are wind, temperature, pressure, density, etc. changing based on what they are currently"
- Conservation of mass, momentum and energy in a continuum
- Often approximated in various ways based on assumptions about the phenomena that is being modeled
- Fundamental in a wide variety of fields

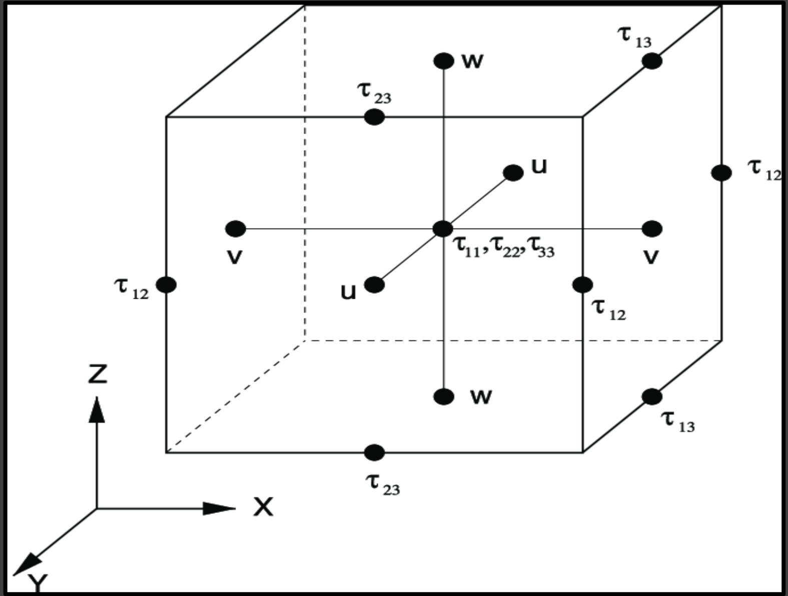




Numerically Solving the PDEs

- Analytic solutions exist only for highly-idealized conditions
- Must be solved numerically for any real-world conditions
- Solved by discretizing over a grid/mesh and approximating the PDE to step forward in time





Flux-Form Euler Equations in WRF

- Excerpt from WRF v4 Documentation:

Using the variables defined above, the flux-form Euler equations can be written as

Wind components

$$\partial_t U + (\nabla \cdot \mathbf{V}u) + \mu_d \alpha \partial_x p + (\alpha/\alpha_d)\partial_\eta p \partial_x \phi = F_U$$

$$\partial_t V + (\nabla \cdot \mathbf{V}v) + \mu_d \alpha \partial_y p + (\alpha/\alpha_d)\partial_\eta p \partial_y \phi = F_V$$

$$\partial_t W + (\nabla \cdot \mathbf{V}w) - g[(\alpha/\alpha_d)\partial_\eta p - \mu_d] = F_W$$

geopotential, moisture, etc.

$$\partial_t \Theta_m + (\nabla \cdot \mathbf{V}\theta_m) = F_{\Theta_m}$$

$$\partial_t \mu_d + (\nabla \cdot \mathbf{V}) = 0$$

$$\partial_t \phi + \mu_d^{-1}[(\mathbf{V} \cdot \nabla \phi) - gW] = 0$$

$$\partial_t Q_m + (\nabla \cdot \mathbf{V}q_m) = F_{Q_m}$$

Numerical Solution to the PDE

$$
\begin{aligned}
\partial_t U + (\nabla \cdot \mathbf{V} u) + \mu_d \alpha \partial_x p + (\alpha/\alpha_d)\partial_\eta p \partial_x \phi &= F_U \\
\partial_t V + (\nabla \cdot \mathbf{V} v) + \mu_d \alpha \partial_y p + (\alpha/\alpha_d)\partial_\eta p \partial_y \phi &= F_V \\
\partial_t W + (\nabla \cdot \mathbf{V} w) - g[(\alpha/\alpha_d)\partial_\eta p - \mu_d] &= F_W \\
\partial_t \Theta_m + (\nabla \cdot \mathbf{V} \theta_m) &= F_{\Theta_m} \\
\partial_t \mu_d + (\nabla \cdot \mathbf{V}) &= 0 \\
\partial_t \phi + \mu_d^{-1}[(\mathbf{V} \cdot \nabla \phi) - gW] &= 0 \\
\partial_t Q_m + (\nabla \cdot \mathbf{V} q_m) &= F_{Q_m}
\end{aligned}
$$

- Note that each of the prognostic variables $U, V, W, \Theta_m, \mu_d, \phi, Q_m$ is known at time t_0.
- How do we determine what these variables are in the future? (i.e. at time t_N)

TempoQuest
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Numerical Solution to the PDE

- Terms in RED are either known or can be approximated using finite difference methods (notice only spatial derivatives)
- Terms in BLUE describe how each variable is changing with respect to time

$$
\begin{aligned}
\partial_t U + (\nabla \cdot \mathbf{V}u) + \mu_d \alpha \partial_x p + (\alpha/\alpha_d)\partial_\eta p \partial_x \phi &= F_U \\
\partial_t V + (\nabla \cdot \mathbf{V}v) + \mu_d \alpha \partial_y p + (\alpha/\alpha_d)\partial_\eta p \partial_y \phi &= F_V \\
\partial_t W + (\nabla \cdot \mathbf{V}w) - g[(\alpha/\alpha_d)\partial_\eta p - \mu_d] &= F_W \\
\partial_t \Theta_m + (\nabla \cdot \mathbf{V}\theta_m) &= F_{\Theta_m} \\
\partial_t \mu_d + (\nabla \cdot \mathbf{V}) &= 0 \\
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Numerical Solution to the PDE

- Terms in RED are either known or can be approximated using finite difference methods (notice only spatial derivatives)
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$$\partial_t U + (\nabla \cdot \mathbf{V}u) + \mu_d \alpha \partial_x p + (\alpha/\alpha_d)\partial_\eta p \partial_x \phi = F_U$$
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$$\partial_t \mu_d + (\nabla \cdot \mathbf{V}) = 0$$
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$$\partial_t Q_m + (\nabla \cdot \mathbf{V}q_m) = F_{Q_m}$$

For each of the the prognostic variables (will use U for this example):

1. Solve for and approximate the *time derivative term* $\partial_t U(t_i)$
2. *For small time intervals (Δt) we can approximate $U(t_{i+1}) \approx U(t_i) + \partial_t U(t_i)\Delta t$

* To simplify the explanation, we use forward Euler but note that WRF uses RK3 for forward time stepping

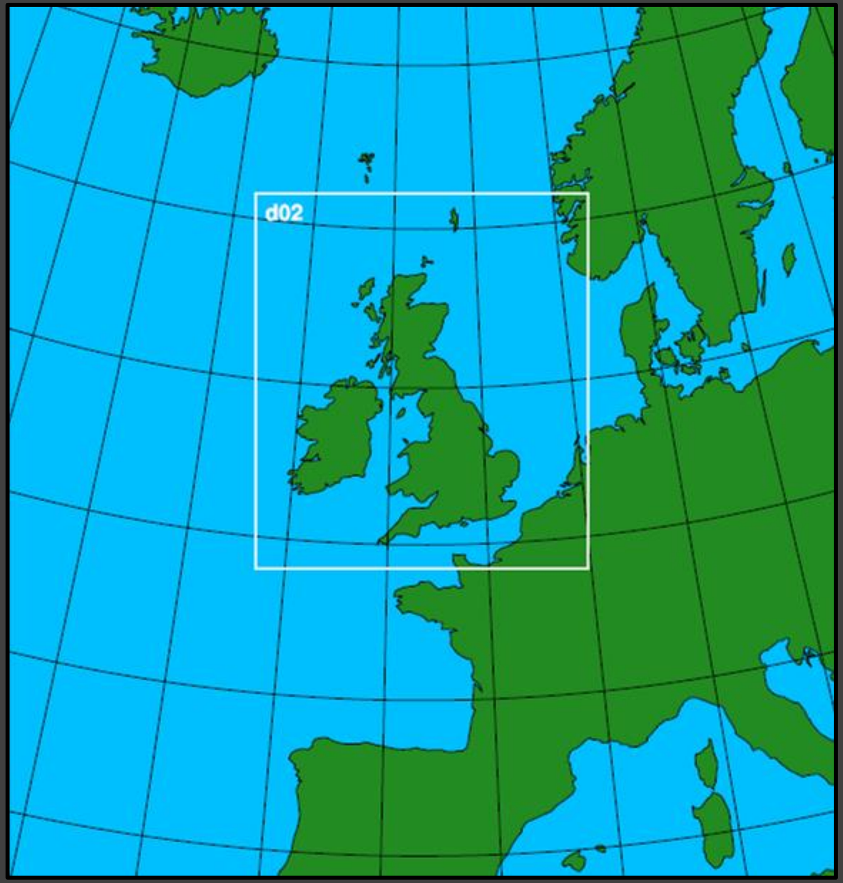
NWP Model Initial/Boundary Conditions

Initial Conditions:

- User-designed idealized fields
- Observations (Data Assimilation)
- Previous simulation results

Boundary Conditions:

- Variety of methods used for vertical BCs
  - Global models: periodic horizontally
    - No horizontal BCs needed!
  - Regional models: require external simulation output encompassing the domain both geographically and temporally



NWP Model Physics

- Capture average effects of sub-grid-scale processes